

IMPROVEMENTS OF SPACIAL RESOLUTION OF CORLORED-GRID BACKGROUND ORIENTED SCHLIEREN (CGBOS) TECHNIQUE BY INTRODUCING TELECENTRIC OPTICAL SYSTEM AND RECONSTRUCTION OF DENSITY FIELD

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ABSTRACT : The background oriented schlieren (BOS) technique is one of the visualization techniques that enable the quantitative measurement of density information in the flow field with very simple experimental setup. The principle of BOS is similar to conventional Schlieren technique, it exploit the bending of light caused by refractive index change corresponding to density change in the medium and both techniques are sensible to density gradient. In our previous study, we have proposed the Colored Grid Background Oriented Schlieren (CGBOS) technique using a colored grid pattern for a background image. In this report we propose a novel approach for BOS technique by introducing telecentric optical system to improve the spatial resolution of BOS measurement. The experiments were carried out in the 0.6 m \times 0.6 m test section of supersonic wind tunnel at JAXA-ISAS. The advantages of telecentric CGBOS are discussed by comparison with normal CGBOS result. Furthermore reconstruction of density field obtained by telecentric CGBOS and Computed Tomography (CT) technique is reported.

1 Introduction

The Background Oriented Schlieren (BOS) technique is based on a patent by Meier [1] and described by Richard [2] and Meier [3]. BOS technique enables us to have the quantitative density measurement with computer-aided image analysis. In the past several years, BOS technique had applied to various experiments - wind tunnel experiment [2]~[5], free flight experiment [5], free jet [5], [6], rotor blade tip vortex of full-scale helicopter [7], etc. The sensitivity and accuracy of BOS is examined by Goldhahn and Seume [6]. Recently Venkatakrishnan and Suriyanarayanan reported precise measurement of 3D density field of separated flow by BOS [8]. The principle of BOS is similar to conventional Schlieren technique, it exploit the bending of light caused by refractive index change corresponding to density change in the medium and both techniques are sensible to density gradient. Conventional Schlieren technique employs many optical elements - pinhole, concave mirror, knife edge or color filter, camera...etc, however it is difficult to realize quantitative measurement and this technique is commonly

used for qualitative measurement like flow visualization. On the other hand BOS requires only a background and a digital still camera and it can realize the quantitative measurement of density. Figure 1 shows optical setup for BOS technique [5]. If there is density change between the background and camera, background image is captured at CMOS sensor of digital still camera with displacement Δh because of the refraction of the light passing through density gradient as shown as a solid line. The relation between Δh and refractive index n is expressed as equation 1 where l_b denotes the distance from background to phase object, l_c the distance from phase object to camera, f the focal length of camera, *n* the refractive index and ε deflection angle [4]. The relation between density ρ and refractive index *n* is given by the Gladstone-Dale equation expressed as equation 2 where *K* is the Gladstone-Dale constant. The integration of spatial gradient of refractive index along light pass can be obtained from equation 1 by calculation of displacement Δh with image analysis. The density information can be also determined with equation 2. Most of BOS techniques that have been applied to laboratory measurement employ monochromatic or colored random dot pattern as a background image. In these measurement two images are required - reference and test image. The reference image is generated by recording the background under no-flow condition before or after the experiment. The test image is recorded under flow condition and it contains disturbance with flow that causes the displacement of background. The displacement of dot pattern is calculated by comparing both images with cross-correlation algorithm commonly used in PIV (Particle Image Velocimetry) technique. On the other hand, horizontal stripe was employed for background image in 'synthetic schlieren' [9], [10]. Dalzeil et al applied synthetic schlieren to measure the gradient of the perturbation density by line refractometry using horizontal lines, dot tracking refractmetry using regular array of dots, and pattern matching refractmetry using randomized array of dots for background [9]. Onu et al adapted synthetic schlieren to measure the vertical amplitude of axisymmetric internal waves by using horizontal lines for background [10]. They also obtained internal wave amplitude in plane of interest with inverse tomographic technique. In these reports, the displacement of line pattern was calculated by taking difference between two images in contrast to calculating the center position of lines directly with finite-fringe analysis method explained later.

In this paper a novel approach for BOS technique is proposed. Telecentric optical system is introduced to the BOS measurement to aim the improvements in the spatial resolution. Figure 2 shows telecentric optical system using two lenses and an aperture. An aperture is placed in the focal point of the two lenses. Parallel light ray can be corrected by telecentric optical system as shown in Fig. 2. This arrangement is called double-sided telecentric lens [11]. The advantages of this optical system are the depth of focus is much longer than normal lenses, and the image magnification does not depend on the distance between the object and the lens because parallel projection is obtained by the telecentric



Fig. 1 Optical setup of BOS



Fig. 2 Telecentric optical systems

$$\Delta h = \frac{l_b f}{l_b + l_c - f} \frac{1}{n_0} \int_{l_b - \Delta l_b}^{l_b + \Delta l_b} \frac{\partial n}{\partial r} dl \tag{1}$$

$$\frac{n-1}{K} = \rho \tag{2}$$

optical system. We have to focus on the background in BOS measurement, while observed phenomena is placed between background and camera. If the view angle is large when normal lenses are used, the observed phenomena can be blurred. This problem can be overcome by introducing telecentric optical system. The disadvantage of telecentric optical system is the obtained image become darker than normal lenses because they capture only a parallel light ray, and the maximum size of the field view is restricted by the size of the entry lens. These disadvantages could spoil the advantage of BOS measurements, however more precise measurement with BOS technique can be achieved by introducing telecentric optical system. Furthermore correcting parallel light is important for reconstruction of density because the reconstruction algorithm is composed on the assumption that projection data are obtained with parallel light ray.

2 Experiments

The experiments were carried out in supersonic wind tunnel at JAXA/ISAS, which has 60 cm x 60 cm test section. Figure 3 indicates a schematic diagram of an asymmetric body installed in the supersonic wind tunnel. The measurement system consists of metal halide lamp (continuous), background, and digital still camera (EOS Kiss Digital X) which has 3880×2690 pixels CMOS censor. For the normal BOS measurement, the distance l_b and l_c are set to 710 mm and 3820 mm as shown in Fig. 4. The focal length of camera *f* is 320mm and shutter speed is set to 1/80 second. Thus the mean density field of supersonic flow is captured. Mach number of supersonic flow is set to 2.0. Figure 5 shows the optical



Fig. 3 Schematic diagram of an asymmetric model

Fig. 4 Experimental setup for normal BOS



Fig. 5 Experimental setup for telecentric BOS

setup for telecentric BOS measurement. The setup consists of half of schlieren system and the difference is installing an aperture at focal point of concave mirror instead of the knife-edge. The diameter of concave mirror is 60 cm and its focal length is 600 cm. The shutter speed of camera is set to 1/60 second.

To obtain the multi-directional projection data for reconstruction, CGBOS images were obtained from nineteen projection angles from 0 degree to 90 degree with 5 degree intervals considering the symmetric nature of flow field for both normal BOS and telecentric BOS measurement.

3 Image Processing

Our research group has developed Laser Interferometric Computed Tomography (LICT) technique and succeeded to elucidate three-dimensional (3-D) unsteady and high-speed flow field behind discharging shock waves [12]~[15]. LICT technique employs Mach-Zehnder interferometer and N_2 pulsed laser as a light source to obtain the finite-fringe interferomgram, which represents projection image of flow field. The finite-fringe analysis method suitable for LICT measurement has also developed in our previous study. In this method the center of each fringe is calculated firstly. Secondary, the displacement of fringe pattern at designated position is calculated to obtain the projection data of density information at the position by comparing with fringe pattern at no-flow area. The projection data of whole flow field are obtained by calculating this projection data at all designated sections. Finally, quantitative 3-D density distribution of whole flow field can be reconstructed from multi directional projection data set obtained by this process.

We are proposing Colored Grid Background Oriented Schlieren (CGBOS) technique using coloredgrid background [16]. The two-dimensional (2-D) projection image of this colored-grid background exposed under flow condition supplies density gradient in vertical and horizontal direction and it can be obtained from only one test image. If projected image contains no disturbance area or distortion of field of view due to camera lens is negligible, CGBOS technique does not require the reference image under no-flow condition. Figure 6 shows CGBOS image taken through Mach 2.0 flow with normal BOS setup. In this report, colored background is composed of green and red stripes. The green stripe is used IMPROVEMENTS OF SPACIAL RESOLUTION OF CGBOS TECHNIQUE BY TELECENTRIC OPTICAL SYSTEM AND RECONSTRUCTION OF DENSITY FIELD







Fig. 7 Green channel of CGBOS image



Fig. 8 Red channel of CGBOS image

for horizontal stripe and red stripe for vertical. The distortion of background image along the shock wave and expansion fan is captured. CGBOS image can be separated into green (horizontal) and red (vertical) stripe image by color information. The distortion of background image in vertical direction is obtained from horizontal green-stripe and horizontal distortion is obtained from vertical red-stripe. Figure 7 indicates horizontal green-stripe image from Fig. 6. The displacement of each stripe pattern in vertical and horizontal direction can be obtained with same finite-fringe analysis technique of LICT measurement mentioned above.

4 Reconstruction

In CGBOS technique distributions of displacement of background image (Δh) are obtained quantitatively. Therefore multi-directional CGBOS image were taken from 19 projection angles between 0 degree and 90 degree with 5 degree intervals considering symmetric nature of flow field. The projection data were obtained from 19 CGBOS images that supply separated 19 horizontal greenstripe images and 19 vertical red-stripe images. Three-dimensional density distribution of supersonic flow field around an asymmetric model is reconstructed from these 38 green and red projected images. In this paper, ART (Algebraic Reconstruction Technique) was employed for reconstruction.

The relation of projection angle θ and image plane is illustrated in Fig. 9 [16]. The vertical direction on projected plane is denoted by *r* as shown in this figure and Fig. 6. The gradient of refractive index in *r* direction is obtained from horizontal stripe image (Fig. 7) separated from CGBOS image. The gradient of *n* in *x* and *y* direction $(\partial n/\partial x \text{ and } \partial n/\partial y)$ can be obtained with trigonometric relation between *y*, *z*, and *r* as shown in Fig. 9. The gradient of *n* in *z* direction $(\partial n/\partial z)$ is obtained from vertical stripe image (Fig. 8). Three-dimensional density distribution is determined same procedure with our previous report [16]. Three-dimensional distribution of refractive index gradient in each direction $(\partial n/\partial x, \partial n/\partial y, \text{ and } \partial n/\partial z)$ are obtained by ART reconstruction, then three-dimensional distribution of n(x, y, z) is determined by solving Poisson equation expressed in Eq. 3 using the Successive Over Relaxation method. Finally, normalized density distribution (ρ/ρ_0) is obtained by the relation between *n* and ρ expressed in Eq. 4 and Eq. 5. In Eq. 3 *S* is obtained by calculating the gradient of reconstructed distributions of $\partial n/\partial x$, $\partial n/\partial y$, and $\partial n/\partial z$.



$$\frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2} + \frac{\partial^2 n}{\partial z^2} = S$$
(3)

$$\rho_0 K = n_0 - 1 \tag{4}$$

$$\frac{\rho}{\rho_0} = \frac{n-1}{n_0 - 1}$$
(5)

Fig. 9 Projection angle and image plane

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5 Results

The 8-bit gray-scale image of calculated displacement for normal CGBOS and telecentric CGBOS measurement are shown in Fig. 10. Black and white color represents shift in lower and upper direction in pixels as shown with the legend in each image. Gray level is set to same range for a comparison between normal and telecentric optical system. These images represent density gradient in vertical direction and equivalent to conventional schlieren image taken with horizontal knife-edge (shown in Fig. 11). Bow shock generated from the tip of asymmetric model and expansion fan from inflection points of model are captured clearly. Both images are obtained from only one projected CGBOS image without averaging multi-exposed BOS images and they represent enough signal to noise ratio for reconstruction. Captured shockwave from normal CGBOS seems to be thicker than conventional schlieren, this is caused by the focal gap between background and test model and observed phenomena could be blurred. On the other hand shockwave is captured sharply with telecentric CGBOS measurement and some disturbances generated from model surface is captured. This is a benefit of telecentricity of observation and the resolution of obtained image is improved. Plot of vertical displacement for both normal and telecentric CGBOS are indicated in Fig. 12. The position is indicated in left image of Fig. 10 with line A-A'. Comparing to normal CGBOS measurement, shock wave and expansion wave is captured more sharply with telecentric CGBOS measurement.

Reconstructed density distribution and isopycnic surface ($\rho/\rho_0 = 0.7$) is illustrated in Fig. 13 and Fig. 14. Reconstruction and calculation of density distribution are done with same procedure for both measurements. Density distributions on *y*-*z* plane seem to be similar, however low-density regions indicated with isopycnic surface shows difference.



Fig. 10 Calculated vertical displacement of horizontal (green) stripe obtained with normal CGBOS (left) and telecentric CGBOS setup (right)



Fig. 11 Schlieren image (horizontal knife edge)

Fig. 12 Plots of vertical displacement on Fig. 10



Fig. 13 Reconstructed density distribution on *y*-*z* plane and isopycnic surface ($\rho/\rho_0 = 0.7$) for normal CGBOS (left) and telecentric CGBOS measurement



Fig. 14 Bird view of isopycnic surface ($\rho/\rho_0=0.7$) and pseudo-color of density distribution on y-z plane for normal CGBOS (left) and telecentric CGBOS (right)

6 Conclusion

The CGBOS technique using colored-grid background is proposed. The displacements of background in r and z direction on projected image were able to obtained from only one test image. The colored grid background was separated into horizontal and vertical stripes based on color information. The finite-fringe analysis technique was applied for calculation of displacement of separated stripe-patterns. The resultant images of displacement in vertical and horizontal direction supply good signal to noise ratio without averaging multi-exposures. Telecentric optical systems are applied to CGBOS technique to improve the spatial resolution of BOS measurement. As a result, displacement data obtained from projected CGBOS image can supply better quality than normal BOS result. Phenomena in flow field, shock wave, expansion wave, etc. can be captured clearly by telecentric BOS setup.

Three-dimensional distribution of refractive index was reconstructed by ART and density distribution in an asymmetrical flow field was obtained quantitatively. Asymmetrical flow phenomena around an asymmetric model in Mach 2.0 flow are captured in detail. Also the reconstructed density information is improved by introducing telecentric optical system. Telecentric BOS technique will be effective for the measurement of more complex flow phenomena.

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